# Elementary Integration

**1.** **(a)** $-\frac{x}{2}-\frac{1}{4}\ln|2x-1|+C$ **(b)** $\frac{1}{4}\ln|2x-1|-\frac{x}{2}+C$ **(c)** $\ln|2x-1| - x + C$

**(d)** $\frac{xq}{p^2}+\frac{x^2}{2p}-\frac{q^2\ln|px-q|}{p^3}+C$ **(e)** $\frac{ax}{c}+\frac{(bc-ad)\ln|d+cx|}{c^2}+C$ **(f)** $\frac{(3x+2)^3}{9}+C$

**(g)** $\frac{1}{4}(3x+1)^{4/3}+C$ **(h)** $\frac{1}{3}(x^2+3)^{3/2}+C$ **(i)** $5\sqrt{2x+3}+C$

**(j)** $-\frac{1}{2}\sqrt{1-2t^2}+C$ **(k)** $\frac{1}{2}\sqrt{2x^2+5}\left(\frac{2x^4}{5}+\frac{x^2}{3}-\frac{5}{3}\right)+C$ **(l)** $\frac{2}{3(n+1)q}(qx^{n+1}+p)^{3/2}+C$

**(m)** $\sqrt{(x+1)^2+4}+C$ **(n)** $\frac{1}{3}(2t-11)\sqrt{2t+1}+C$ **(o)** $x+ln|x-2|-ln|x+2|+C$

**(p)** $\frac{1}{22}(2x-3)^{11}+C$ **(q)** $\frac{\tan^{11}x}{11}+C$ **(r)** $\frac{1}{2\alpha(n+1)}(\alpha x^2+\beta)^{n+1}+C$ **(s)** $\sqrt{x^2-4}+2\tan^{-1}\left(\frac{\sqrt{x^2-4}}{2}\right)+C$

**(t)** $-2\cos\sqrt{x}+C$ **(u)** $\frac{1}{6600}$ **(v)** $0$ **(w)** $\begin{cases} 0 & ,m\in 2\mathbf{Z} \\ \frac{1}{2m} & ,m\in 2\mathbf{Z}-1 \end{cases}$

**2.** **(a)** 1 **(b)** 1

**3.** **(a)** 0 **(b)** $-\sin(a^2)$ **(c)** $\sin(h^2)$

**4.** **(a)** $\frac{1}{3}-\ln 2$ **(b)** $\frac{45}{4}$

**5.** $-|f(x)|\le f(x)\le |f(x)|\quad \forall x\in[a,b] \Rightarrow -\int_a^b |f(x)|dx\le\int_a^b f(x)dx\le\int_a^b |f(x)|dx \Rightarrow \left|\int_a^b f(x)dx\right|\le\int_a^b |f(x)|dx$

**6.** **(a)** $2x\sqrt{1+x^4}$ **(b)** $\frac{3x^2}{\sqrt{1+x^{12}}}-\frac{2x}{\sqrt{1+x^8}}$ **(c)** $-\sin x\cos[\pi\cos^2 x]-\cos x\cos[\pi\sin^2 x]$

**7.** **(a)** First part, book work.

Since $x^n > 0$ in $[0,1]$, $\int_0^1 \frac{x^n}{1+x}dx=(x_0{}^n)\int_0^1\frac{1}{1+x}dx=(x_0{}^n)\ln(1+x)\Big|_0^1=(x_0{}^n)\ln 2$, where $x_0\in[0,1]$

Since $\lim_{n\to\infty}(x_0{}^n)=0$, $\lim_{n\to\infty}\int_0^1\frac{x^n}{1+x}dx=0$.

**(b)** $x^n=(1+x)x^{n-1}-x^{n-1} \Rightarrow \frac{x^n}{1+x}=x^{n-1}-\frac{x^{n-1}}{1+x} \Rightarrow \int_0^1\frac{x^n}{1+x}dx=\int_0^1 x^{n-1}dx-\int_0^1\frac{x^{n-1}}{1+x}dx$

$$\Rightarrow\int_0^1\frac{x^n}{1+x}dx=\frac{1}{n}-\int_0^1\frac{x^{n-1}}{1+x}dx$$

**(c)** Replace $n$ by $1, -2, 3, -4,\ldots, (-1)^{n-1}n$ in **(b)**, we get n equalities.

Add these $n$ equalities up and put $n\to\infty$.

By **(a)**, all involving integrals are 0, except the integral : $\int_0^1\frac{1}{1+x}dx=\ln 2$.

Therefore $\lim_{n\to\infty}\left[1-\frac{1}{2}+\frac{1}{3}-\frac{1}{4}+\ldots+(-1)^{n-1}\frac{1}{n}\right]=\ln 2$.